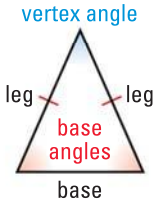
4.7 Use Isosceles and Equilateral Triangles

Before You learned about isosceles and equilateral triangles.
Now You will use theorems about isosceles and equilateral triangles.
Why? So you can solve a problem about architecture, as in Ex. 40.

Key Vocabulary
- **legs**
- **vertex angle**
- **base**
- **base angles**

In Lesson 4.1, you learned that a triangle is isosceles if it has at least two congruent sides. When an isosceles triangle has exactly two congruent sides, these two sides are the **legs**. The angle formed by the legs is the **vertex angle**. The third side is the **base** of the isosceles triangle. The two angles adjacent to the base are called **base angles**.



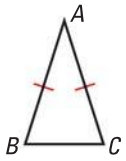
THEOREMS *For Your Notebook*

THEOREM 4.7 Base Angles Theorem

If two sides of a triangle are congruent, then the angles opposite them are congruent.

If $\overline{AB} \cong \overline{AC}$, then $\angle B \cong \angle C$.

Proof: p. 265

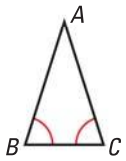


THEOREM 4.8 Converse of Base Angles Theorem

If two angles of a triangle are congruent, then the sides opposite them are congruent.

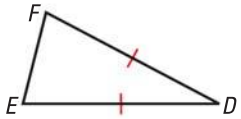
If $\angle B \cong \angle C$, then $\overline{AB} \cong \overline{AC}$.

Proof: Ex. 45, p. 269



EXAMPLE 1 Apply the Base Angles Theorem

In $\triangle DEF$, $\overline{DE} \cong \overline{DF}$. Name two congruent angles.



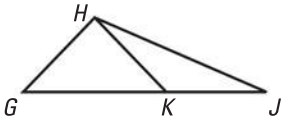
Solution

▶ $\overline{DE} \cong \overline{DF}$, so by the Base Angles Theorem, $\angle E \cong \angle F$.

✓ GUIDED PRACTICE for Example 1

Copy and complete the statement.

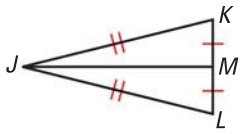
1. If $\overline{HG} \cong \overline{HK}$, then $\angle$? $\cong \angle$?.
2. If $\angle KHJ \cong \angle KJH$, then ? $\cong$?.



PROOF Base Angles Theorem

GIVEN ▶ $\overline{JK} \cong \overline{JL}$

PROVE ▶ $\angle K \cong \angle L$



Plan for Proof

a. Draw $\overline{JM}$ so that it bisects $\overline{KL}$.

b. Use SSS to show that $\triangle JMK \cong \triangle JML$.

c. Use properties of congruent triangles to show that $\angle K \cong \angle L$.

Plan in Action

	STATEMENTS	REASONS
	1. M is the midpoint of $\overline{KL}$.	1. Definition of midpoint
a.	2. Draw $\overline{JM}$.	2. Two points determine a line.
	3. $\overline{MK} \cong \overline{ML}$	3. Definition of midpoint
	4. $\overline{JK} \cong \overline{JL}$	4. Given
	5. $\overline{JM} \cong \overline{JM}$	5. Reflexive Property of Congruence
b.	6. $\triangle JMK \cong \triangle JML$	6. SSS Congruence Postulate
c.	7. $\angle K \cong \angle L$	7. Corresp. parts of $\cong$ △ are $\cong$.

Recall that an equilateral triangle has three congruent sides.

WRITE A BICONDITIONAL

The corollaries state that a triangle is *equilateral* if and only if it is *equiangular*.

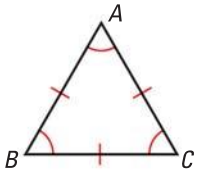
COROLLARIES *For Your Notebook*

Corollary to the Base Angles Theorem

If a triangle is equilateral, then it is equiangular.

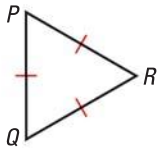
Corollary to the Converse of Base Angles Theorem

If a triangle is equiangular, then it is equilateral.



EXAMPLE 2 Find measures in a triangle

Find the measures of $\angle P$, $\angle Q$, and $\angle R$.



The diagram shows that $\triangle PQR$ is equilateral. Therefore, by the Corollary to the Base Angles Theorem, $\triangle PQR$ is equiangular. So, $m\angle P = m\angle Q = m\angle R$.

$3(m\angle P) = 180^\circ$ **Triangle Sum Theorem**

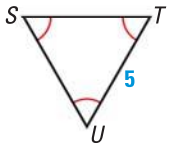
$m\angle P = 60^\circ$ **Divide each side by 3.**

▶ The measures of $\angle P$, $\angle Q$, and $\angle R$ are all 60°.

GUIDED PRACTICE for Example 2

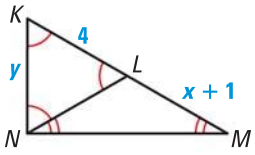
3. Find ST in the triangle at the right.

4. Is it possible for an equilateral triangle to have an angle measure other than 60°? *Explain*.



EXAMPLE 3 Use isosceles and equilateral triangles

ALGEBRA **Find the values of x and y in the diagram.**



Solution

STEP 1 **Find** the value of y. Because $\triangle KLN$ is equiangular, it is also equilateral and $\overline{KN} \cong \overline{KL}$. Therefore, $y = 4$.

AVOID ERRORS
You cannot use $\angle N$ to refer to $\angle LNM$ because three angles have N as their vertex.

STEP 2 **Find** the value of x. Because $\angle LNM \cong \angle LMN$, $\overline{LN} \cong \overline{LM}$ and $\triangle LMN$ is isosceles. You also know that $LN = 4$ because $\triangle KLN$ is equilateral.

$LN = LM$ **Definition of congruent segments**

$4 = x + 1$ **Substitute 4 for *LN* and $x + 1$ for *LM*.**

$3 = x$ **Subtract 1 from each side.**

EXAMPLE 4 Solve a multi-step problem

LIFEGUARD TOWER **In the lifeguard tower, $\overline{PS} \cong \overline{QR}$ and $\angle QPS \cong \angle PQR$.**

a. What congruence postulate can you use to prove that $\triangle QPS \cong \triangle PQR$?

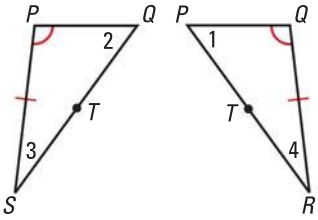
b. Explain why $\triangle PQT$ is isosceles.

c. Show that $\triangle PTS \cong \triangle QTR$.

Solution

AVOID ERRORS
When you redraw the triangles so that they do not overlap, be careful to copy all given information and labels correctly.

a. Draw and label $\triangle QPS$ and $\triangle PQR$ so that they do not overlap. You can see that $\overline{PQ} \cong \overline{QP}$, $\overline{PS} \cong \overline{QR}$, and $\angle QPS \cong \angle PQR$. So, by the SAS Congruence Postulate, $\triangle QPS \cong \triangle PQR$.

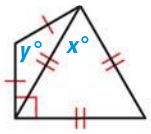


b. From part (a), you know that $\angle 1 \cong \angle 2$ because corresp. parts of $\cong$ $\triangle$ are $\cong$. By the Converse of the Base Angles Theorem, $\overline{PT} \cong \overline{QT}$, and $\triangle PQT$ is isosceles.

c. You know that $\overline{PS} \cong \overline{QR}$, and $\angle 3 \cong \angle 4$ because corresp. parts of $\cong$ $\triangle$ are $\cong$. Also, $\angle PTS \cong \angle QTR$ by the Vertical Angles Congruence Theorem. So, $\triangle PTS \cong \triangle QTR$ by the AAS Congruence Theorem.

GUIDED PRACTICE for Examples 3 and 4

5. Find the values of x and y in the diagram.



6. **REASONING** Use parts (b) and (c) in Example 4 and the SSS Congruence Postulate to give a different proof that $\triangle QPS \cong \triangle PQR$.

4.7 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 17, and 41

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 18, 19, 30, 31, 42, and 46

SKILL PRACTICE

1. **VOCABULARY** Define the *vertex angle* of an isosceles triangle.

2. ★ **WRITING** What is the relationship between the base angles of an isosceles triangle? *Explain*.

EXAMPLE 1 on p. 264 for Exs. 3–6

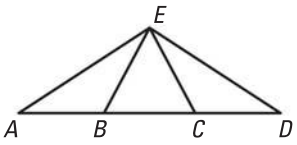
USING DIAGRAMS **In Exercises 3–6, use the diagram. Copy and complete the statement. Tell what theorem you used.**

3. If $\overline{AE} \cong \overline{DE}$, then $\angle$? $\cong \angle$?.

4. If $\overline{AB} \cong \overline{EB}$, then $\angle$? $\cong \angle$?.

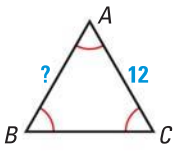
5. If $\angle D \cong \angle CED$, then ? $\cong$?.

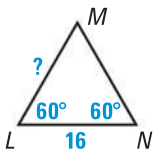
6. If $\angle EBC \cong \angle ECB$, then ? $\cong$?.

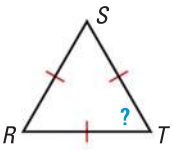


EXAMPLE 2 on p. 265 for Exs. 7–14

REASONING **Find the unknown measure.**

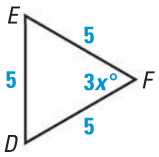
7. 

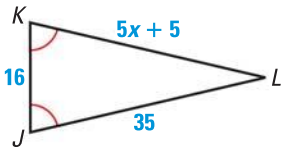
8. 

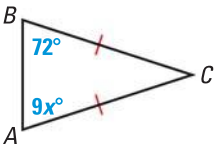
9. 

10. **DRAWING DIAGRAMS** A base angle in an isosceles triangle measures 37°. Draw and label the triangle. What is the measure of the vertex angle?

ALGEBRA **Find the value of x.**

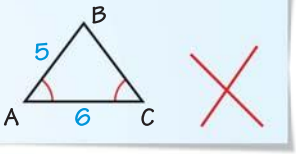
11. 

12. 

13. 

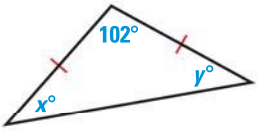
14. **ERROR ANALYSIS** *Describe* and correct the error made in finding BC in the diagram shown.

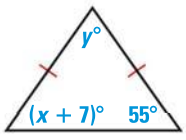
$\angle A \cong \angle C$, therefore $\overline{AC} \cong \overline{BC}$. So, $BC = 6$

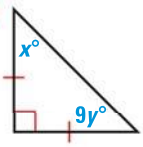


EXAMPLE 3 on p. 266 for Exs. 15–17

ALGEBRA **Find the values of x and y.**

15. 

16. 

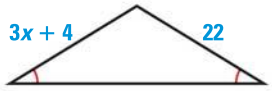
17. 

18. ★ **SHORT RESPONSE** Are isosceles triangles always acute triangles? *Explain* your reasoning.

19. ★ **MULTIPLE CHOICE** What is the value of x in the diagram?

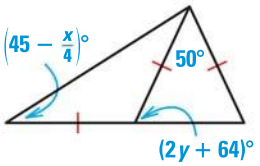
Ⓐ 5 Ⓑ 6

Ⓒ 7 Ⓓ 9



xy ALGEBRA **Find the values of x and y, if possible. *Explain* your reasoning.**

20.



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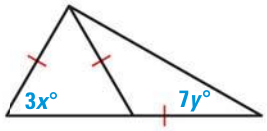
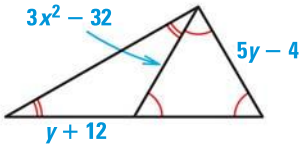
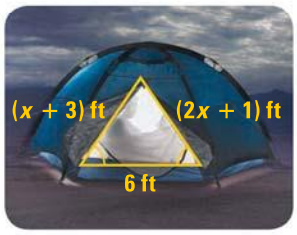


22.



xy ALGEBRA **Find the perimeter of the triangle.**

23.



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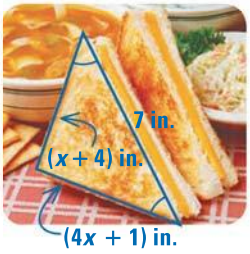

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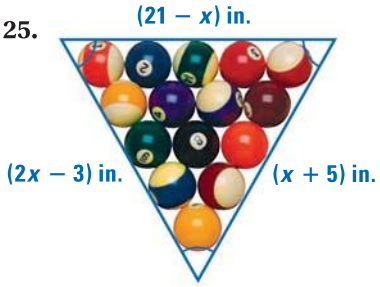


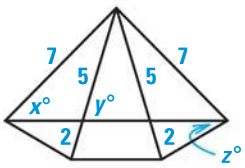
REASONING **In Exercises 26–29, use the diagram. State whether the given values for x, y, and z are possible or not. If not, *explain*.**

26. $x = 90, y = 68, z = 42$

27. $x = 40, y = 72, z = 36$

28. $x = 25, y = 25, z = 15$

29. $x = 42, y = 72, z = 33$

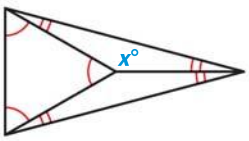


30. ★ **SHORT RESPONSE** In $\triangle DEF$, $m\angle D = (4x + 2)°$, $m\angle E = (6x - 30)°$, and $m\angle F = 3x°$. What type of triangle is $\triangle DEF$? *Explain* your reasoning.

31. ★ **SHORT RESPONSE** In $\triangle ABC$, D is the midpoint of $\overline{AC}$, and $\overline{BD}$ is perpendicular to $\overline{AC}$. *Explain* why $\triangle ABC$ is isosceles.

xy ALGEBRA **Find the value(s) of the variable(s). *Explain* your reasoning.**

32.



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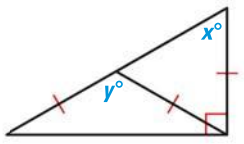
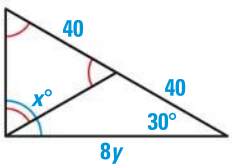


34.



35. **REASONING** The measure of an exterior angle of an isosceles triangle is 130°. What are the possible angle measures of the triangle? *Explain*.

36. **PROOF** Let $\triangle ABC$ be isosceles with vertex angle $\angle A$. Suppose $\angle A$, $\angle B$, and $\angle C$ have integer measures. Prove that $m\angle A$ must be even.

37. **CHALLENGE** The measure of an exterior angle of an isosceles triangle is $x°$. What are the possible angle measures of the triangle in terms of x? *Describe* all the possible values of x.

PROBLEM SOLVING

38. SPORTS The dimensions of a sports pennant are given in the diagram. Find the values of x and y.

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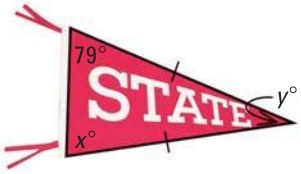

39. ADVERTISING A logo in an advertisement is an equilateral triangle with a side length of 5 centimeters. Sketch the logo and give the measure of each side and angle.

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40. ARCHITECTURE The Transamerica Pyramid building shown in the photograph has four faces shaped like isosceles triangles. The measure of a base angle of one of these triangles is about 85°. What is the approximate measure of the vertex angle of the triangle?

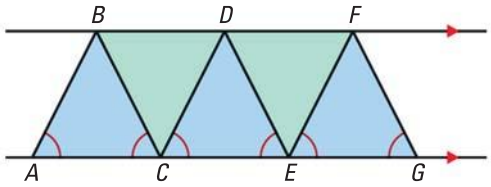
EXAMPLE 4 on p. 266 for Exs. 41–42

41. MULTI-STEP PROBLEM To make a zig-zag pattern, a graphic designer sketches two parallel line segments. Then the designer draws blue and green triangles as shown below.

a. Prove that $\triangle ABC \cong \triangle BCD$.

b. Name all the isosceles triangles in the diagram.

c. Name four angles that are congruent to $\angle ABC$.



42. ★ VISUAL REASONING In the pattern below, each small triangle is an equilateral triangle with an area of 1 square unit.

Triangle				
Area	1 square unit	?	?	?

a. Reasoning *Explain* how you know that any triangle made out of equilateral triangles will be an equilateral triangle.

b. Area Find the areas of the first four triangles in the pattern.

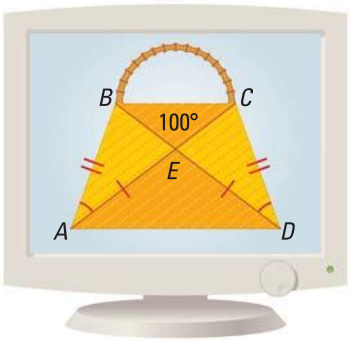
c. Make a Conjecture *Describe* any patterns in the areas. Predict the area of the seventh triangle in the pattern. *Explain* your reasoning.

43. REASONING Let $\triangle PQR$ be an isosceles right triangle with hypotenuse $\overline{QR}$. Find $m\angle P$, $m\angle Q$, and $m\angle R$.

44. REASONING *Explain* how the Corollary to the Base Angles Theorem follows from the Base Angles Theorem.

45. PROVING THEOREM 4.8 Write a proof of the Converse of the Base Angles Theorem.

46. ★ **EXTENDED RESPONSE** Sue is designing fabric purses that she will sell at the school fair. Use the diagram of one of her purses.



a. Prove that $\triangle ABE \cong \triangle DCE$.

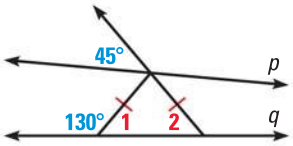
b. Name the isosceles triangles in the purse.

c. Name three angles that are congruent to $\angle EAD$.

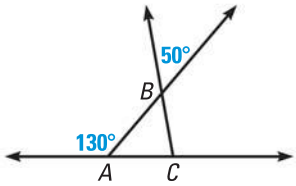
d. **What If?** If the measure of $\angle BEC$ changes, does your answer to part (c) change? *Explain*.

REASONING FROM DIAGRAMS **Use the information in the diagram to answer the question. *Explain* your reasoning.**

47. Is $p \parallel q$?



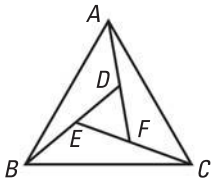
48. Is $\triangle ABC$ isosceles?



49. **PROOF** Write a proof.

GIVEN ▶ $\triangle ABC$ is equilateral, $\angle CAD \cong \angle ABE \cong \angle BCF$.

PROVE ▶ $\triangle DEF$ is equilateral.



50. **COORDINATE GEOMETRY** The coordinates of two vertices of $\triangle TUV$ are $T(0, 4)$ and $U(4, 0)$. *Explain* why the triangle will always be an isosceles triangle if V is any point on the line $y = x$ except $(2, 2)$.

51. **CHALLENGE** The lengths of the sides of a triangle are $3t$, $5t - 12$, and $t + 20$. Find the values of t that make the triangle isosceles. *Explain*.

MIXED REVIEW

What quadrant contains the point? *(p. 878)*

52. $(-1, -3)$ **53.** $(-2, 4)$ **54.** $(5, -2)$

Copy and complete the given function table. *(p. 884)*

55.

x	-7	0	5
$y = x - 4$	?	?	?

56.

?	-2	0	1
?	-6	0	3

PREVIEW
Prepare for Lesson 4.8 in Exs. 57–60.

Use the Distance Formula to decide whether $\overline{AB} \cong \overline{AC}$. *(p. 15)*

57. $A(0, 0)$, $B(-5, -6)$, $C(6, 5)$

58. $A(3, -3)$, $B(0, 1)$, $C(-1, 0)$

59. $A(0, 1)$, $B(4, 7)$, $C(-6, 3)$

60. $A(-3, 0)$, $B(2, 2)$, $C(2, -2)$